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Computing the Pareto front in Multiobjective Linear Mixed Integer Fractional Programming

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Objectives

To present 2 Branch & Bound techniques to compute all the nondominated solutions in multiobjective linear mixed integer fractional programming (MOMILFP).

To compare the techniques through computational experiments.

Notation

$$\max \quad z_k(x) = \frac{\sum_{j=1}^{n} c_{kj}x_j + \alpha_k}{\sum_{j=1}^{n} d_{kj}x_j + \beta_k}, \quad k = 1, \ldots, p$$

$$\text{s. t.: } x \in X = \left\{ \begin{array}{c} x \in R^n | Ax \leq b, A \in R^{m.n} b \in R^m, x \geq 0, \\ x_j \in Z_0^+, j \in J, x_j \in \{0,1\}, j \in I \end{array} \right\}$$

$J, I \subseteq \{1, \ldots, n\}, J \cap I = \emptyset, J \cup I \neq \emptyset$

$c, d \in R^{p.n}; \alpha, \beta \in R^p; d_k x + \beta_k > 0; x \in X, k = 1, \ldots p$

Branch & Bound Techniques

Initialization.

Characterize the nondominated region of the MOMILFP problem by computing its pay-off table.

Branch & Bound process.

i) Select the next nondominated region.

ii) Divide this region into two sub-regions, by imposing constraints on one of the objective functions. 2 approaches:

First technique: Divide by the middle of the objective function biggest range in the pay-off table.

Second technique: Cut a little bit (the predefined error) at the bottom of the region, considering the objective function biggest range in the pay-off table.

iii) Compute the pay-off tables of the two new sub-regions in order to characterize them.

The process is repeated for every sub-region until the remaining sub-regions are ‘smaller’ than a predefined error. One sub-region is ‘smaller’ than a predefined error when the range of values of each objective function in the pay-off table is lower than the predefined error.

An example

$$\begin{array}{l} max \quad z_1(x) = \frac{x_1+3x_2+2}{2x_1+x_2+1} \\ max \quad z_2(x) = \frac{3x_1+x_2+2}{x_1+2x_2+1} \end{array} \text{ s. t.: } \left. \begin{array}{l} x_1, x_2 \in \mathbb{Z}_0^+, \\ 3x_1 + 2x_2 \leq 24 \\ -x_1 + x_2 \leq 4 \\ x_1 - x_2 \leq 4 \end{array} \right\} (x_1, x_2) \in X$$

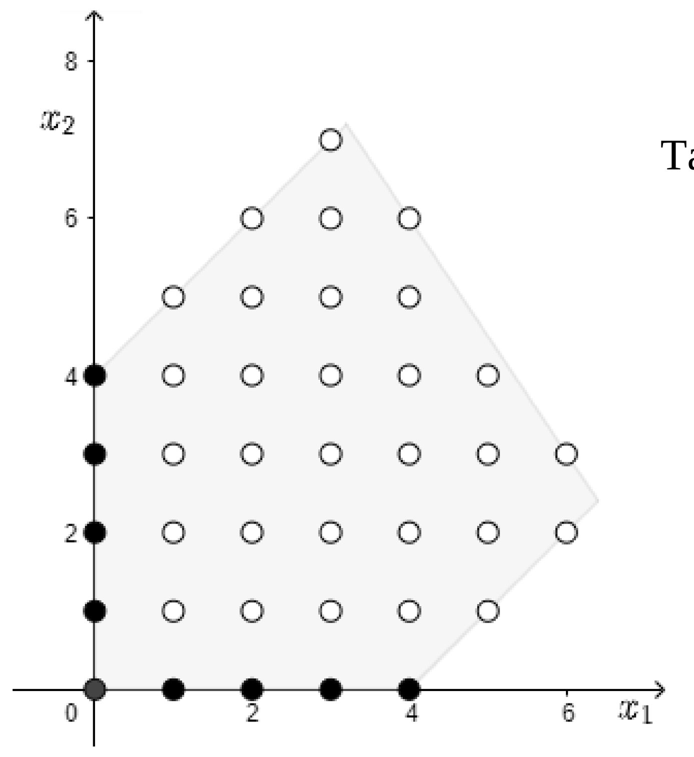


Table 1. All efficient solutions of the example.

x_1	x_2	z_1	z_2	$s(x)$
0	0	2.000	2.000	2.000
0	1	2.500	1.000	1.750
0	2	2.667	0.800	1.733
0	3	2.750	0.714	1.590
0	4	2.800	0.667	1.733
1	0	1.000	2.500	1.750
2	0	0.800	2.667	1.733
3	0	0.714	2.750	1.590
4	0	0.667	2.800	1.733

The techniques provide not only supported but also unsupported nondominated solutions.

It can be observed that the nondominated solution set of a MOLMIFP problem has, in general, a significant part of unsupported solutions.

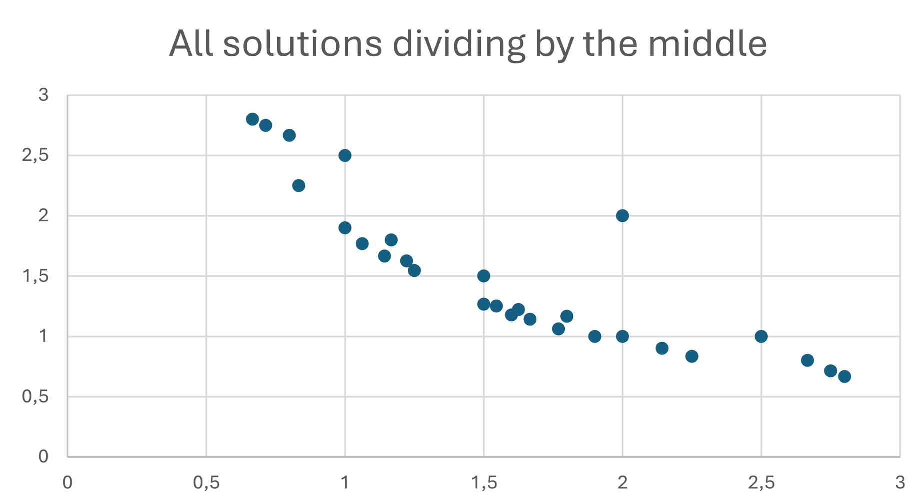
Computational Tests

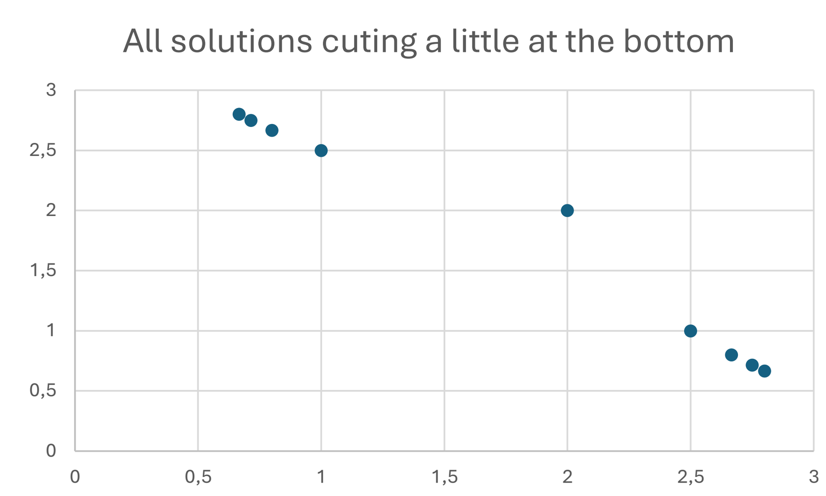
Preliminary tests.

CPLEX™ Optimizer V12.4 for solving the auxiliary linear programming problems.

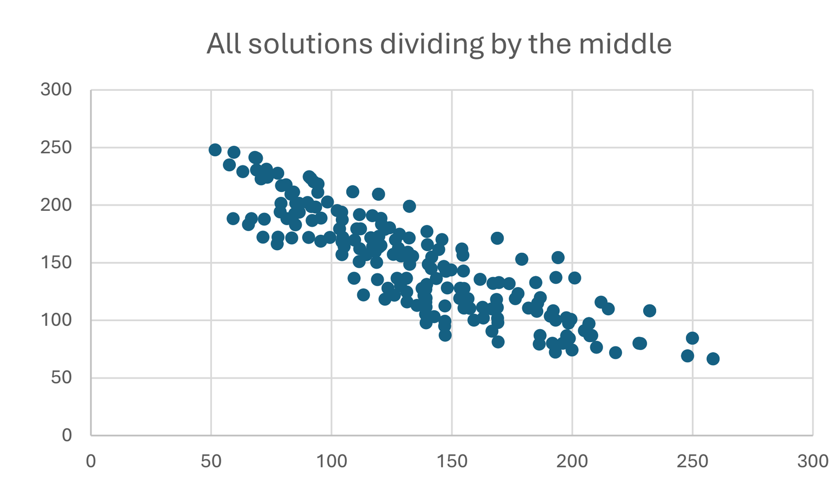
The techniques were coded in Delphi™ XE25 for Win64.

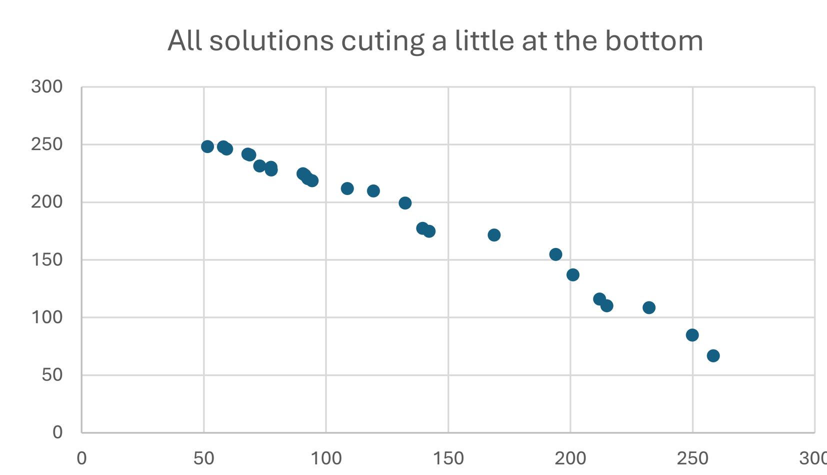
Using the example



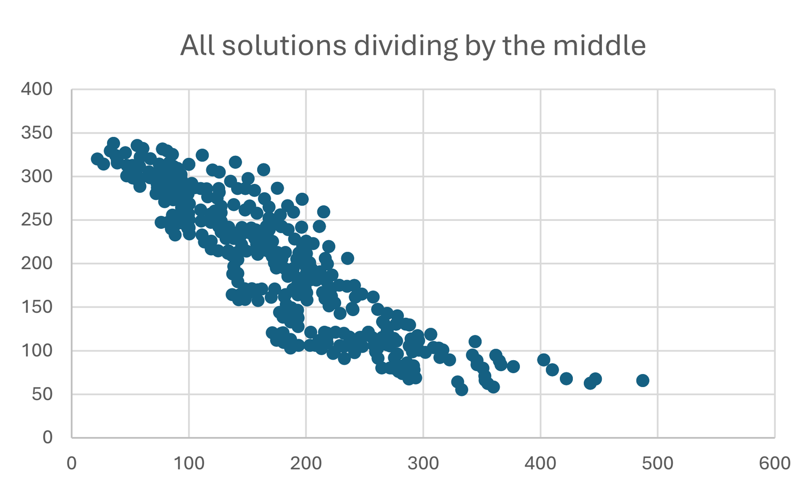


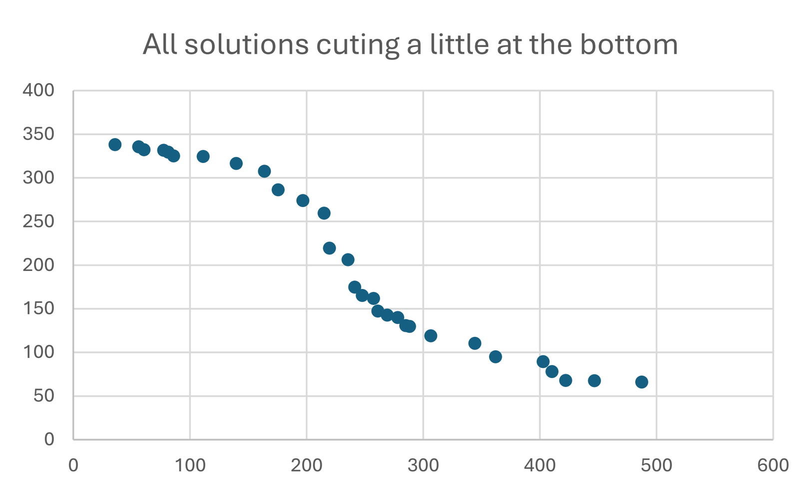
2 objective functions; 5 constraints; 20 binary variables.



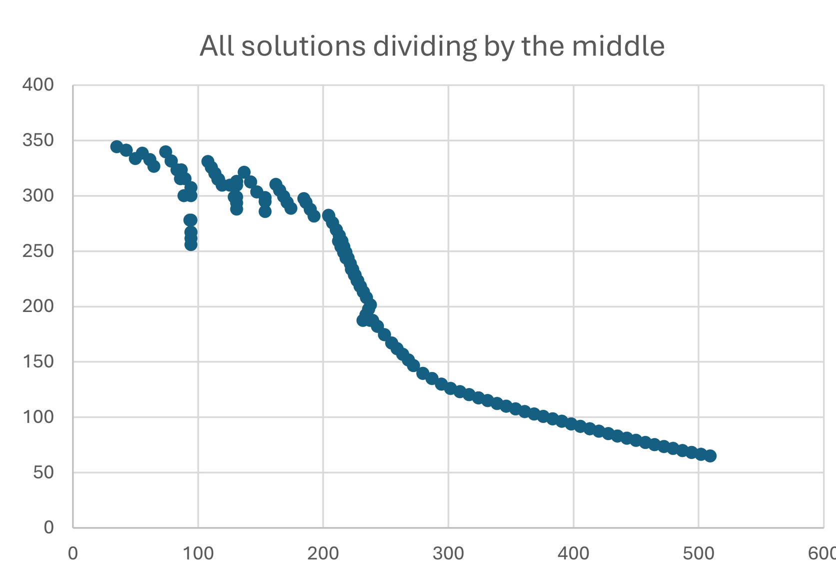


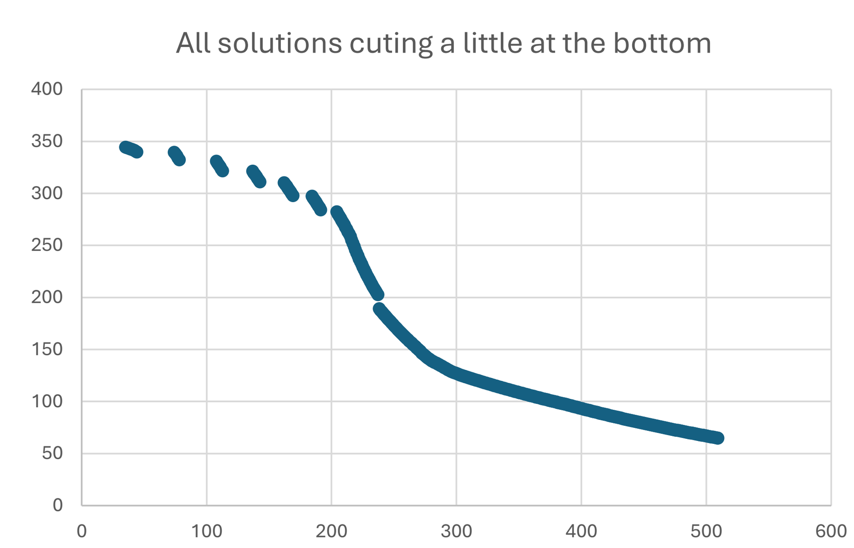
2 objective functions; 5 constraints; 20 integer variables.





2 objective functions; 5 constraints; 10 integer and 10 continuous variables.





Conclusions

Computational experiments show that the algorithms can deal with practical MOMILFP problems with binary, general integer and mixed-integer variables. The second technique seems to be better.

More tests are needed.

The algorithm and the software implementation must be improved to avoid unnecessary computations.