Multicriteria Analysis of Other-Regarding Behavior in Oligopolies with Penalties

Caraballo Pou, Mª Ángeles; Zapata Reina, Asunción; Monroy Berjillos, Luisa; Mármol Conde, Amparo Mª

Introduction

- Penalty: Sanction for deviation from a fixed total quantity (target).
- Several real-life situations: wine sector, fishing sector.

Cournot game with penalties: $\{(E_j, u_j)_{j\in N}\}$

- The firms produce a homogeneous good.
- $P(Q)$: inverse demand function.
- Profit function: $\pi_j(q_1,...,q_n) = q_jP(Q)$.
- $F_j(Q)$: penalty function for firm j when aggregated production deviates from a target γ ($\gamma > 0$).
- F_j twice-continuously differentiable, non-negative, convex. $F_j(\gamma) = 0$ and $F_j'(Q) = 0$ if and only if $Q = \gamma$.
- Utility function: $u_j(q_1,...,q_n) = q_jP(Q) - F_j(Q)$.

Multicriteria game with penalties: $\{(E_j, u)_{j\in N}\}$

- Vector-valued utility function: $u = (u_1, ..., u_n)$.
- Preference function: $v^i(q) = \sum_{j\in N}\lambda^i_j u_j(q), \quad \lambda^i \in \Delta^n$.
- Types of firms with other-regarding (OR) behavior:
  a) equanimous if $\lambda^i_j = \lambda^i_k$ ∀j, k∈ N. b) impartial if $\lambda^i_j = \lambda^i_k$ ∀j, k≠ i.
  c) pro-self if $\lambda^i_i \geq \lambda^i_j$ ∀j∈ N. d) pro-social if $\lambda^i_i \leq \lambda^i_j$ ∀j∈ N.

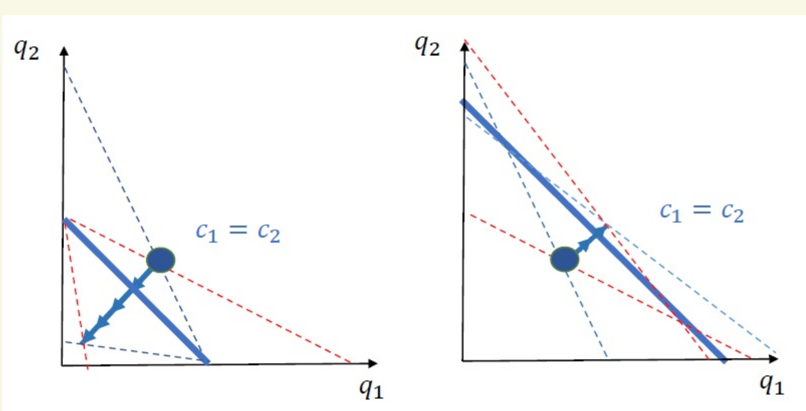
A particular case

- Linear inverse demand function: $P(Q) = a - bQ$.
- Quadratic penalty function: $F_j(Q) = c_j(Q-\gamma)^2$.
- $a, b > 0$ for all $j \in N$, $c_j > 0$: penalty parameter.
- Higher c_j: stronger incentive for firms to align with γ.
- Utility function: $u_j(q_1,...,q_n) = q_j(a - bQ) - c_j(Q-\gamma)^2$.
- Best response: $R^j(q_{-j}) = max\left\{0, \frac{a+2c_j\gamma-(b+2c_j)\sum_{h\neq j} q_h}{2b+2c_j}\right\}$.
- Nash equilibrium: $q_j = \frac{a+2c_j\gamma+2(\frac{a}{b}-\gamma)(C-nc_j)}{(n+1)b+2C}$, with $C = \sum_{h\in N} c_h$.
- OR-Best response, with $\bar{c}_i = \sum_{h\in N}\lambda^i_h c_h$:

$$R^i_\lambda(q_{-i}) = max\{0, \frac{\lambda^i_i a + 2\gamma\lambda^i_i c_i - \sum_{j\neq i}(b(\lambda^i_i+\lambda^i_j)+2\bar{c}_i))q_j}{2b\lambda^i_i + 2\bar{c}_i}\}.$$

Path of equilibria. Firms with the same λ

- Equilibrium exists and is unique except for equanimous firms.
- $\bar{\gamma}$ is the target value such that when $\gamma < \bar{\gamma}$ ($\gamma > \bar{\gamma}$) the total quantity in equilibrium decreases (increases) as λ decreases.
- When firms have the same penalty: $\bar{\gamma} = \frac{a}{b} + \frac{a}{4c}$.

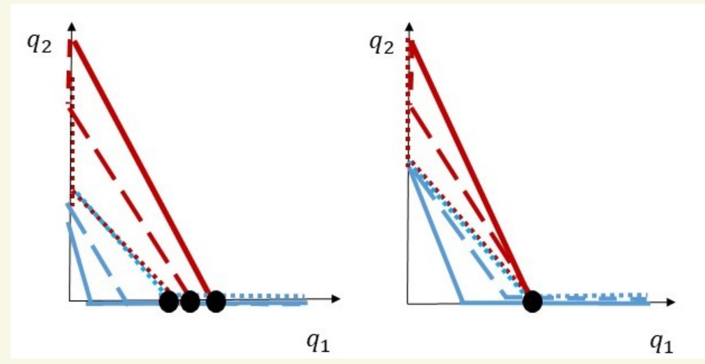


Path of equilibria for identical firms when λ^i varies. Cases $\gamma < \bar{\gamma}$ and $\gamma > \bar{\gamma}$.

Path of equilibria when λ differs

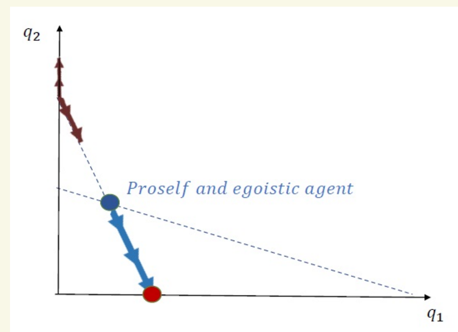
$\hat{\gamma}$ is the target value such that:

- When $\gamma < \hat{\gamma}$ proself firms drive prosocial firms out of the market. When firms have the same penalty: $\hat{\gamma} = \frac{a}{2b}$.

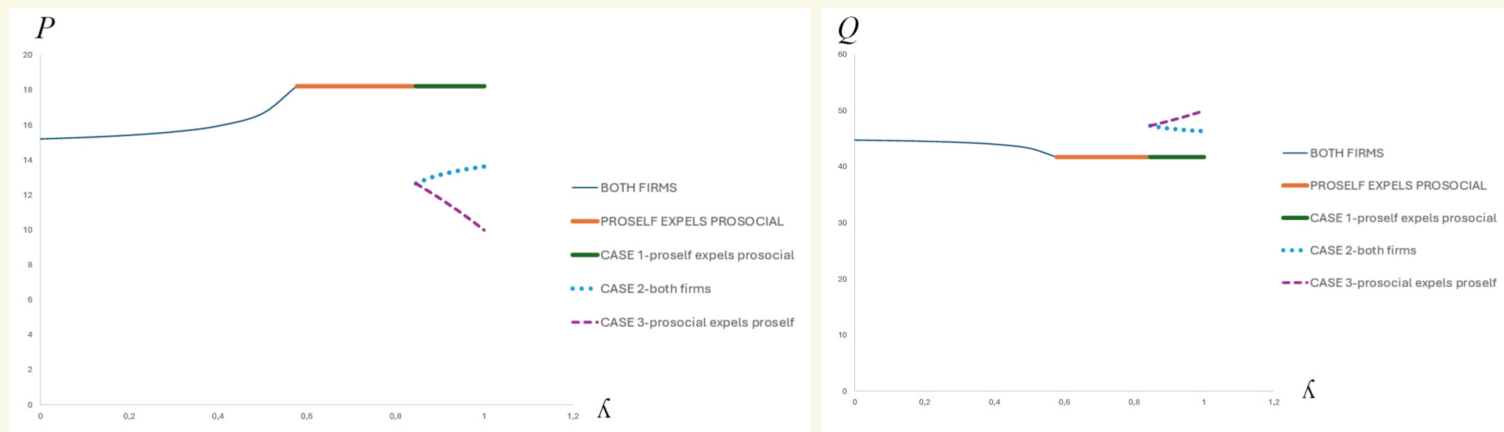


Path of equilibria for different firms ($\gamma < \hat{\gamma}$), *when* λ^i varies.

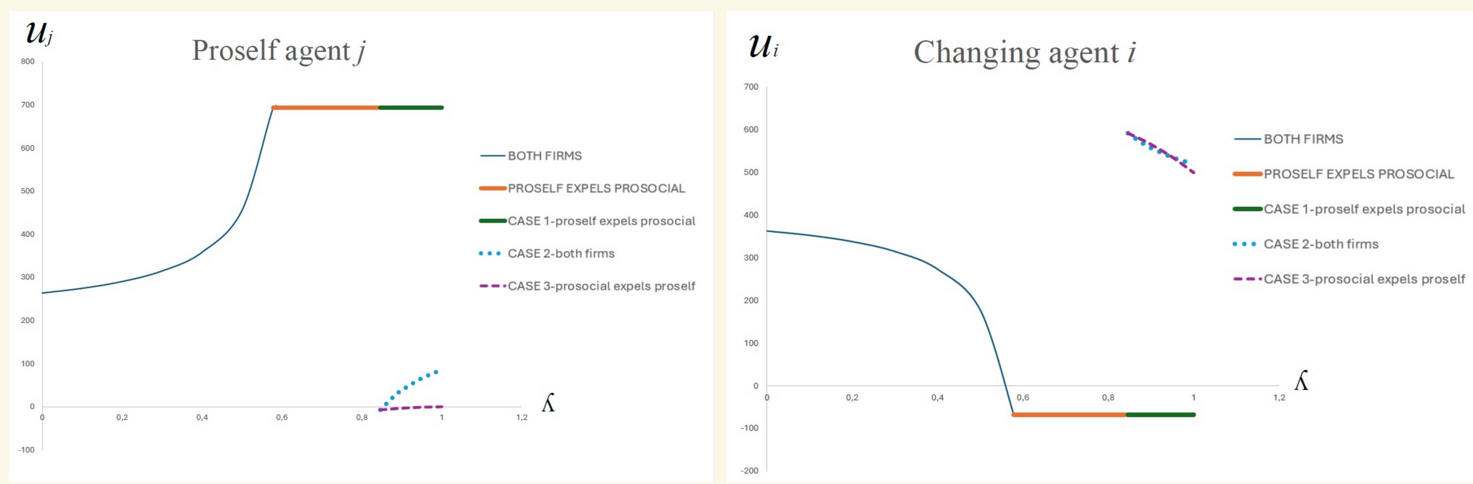
- When $\gamma > \hat{\gamma}$, a discontinuous path of equilibria arises where proself firm expels prosocial firm, prosocial firm expels proself firm and both firms remain.



A discontinuous path of equilibria.
For $P = 60 - Q$, $c_1 = c_2 = 1$, $\gamma = 50$, proself agent j, changing agent i



Price path. Quantity path.



Utility path

Conclusions

Equilibria depend on γ and λ.

- Equilibrium exists when all firms have the same λ.
- Mutiple equilibria: all firms are equanimous.
- When λ differs, equilibria depend both on γ and λ.
  - There are economic sectors where considering other-regarding behavior and quantity goals jointly is relevant.
  - We propose a theoretical framework adaptable to various situations, in which other-regarding behaviour is beneficial for firms. Under some conditions:
    - The most other-regarding firm achieves higher profits than the least other-regarding one and the consumer surplus is also higher.
    - The higher λ_i, the higher the level of quantity produced.

References

Monroy L, Caraballo MA, Mármol AM, Zapata A (2017) Agents with other-regarding preferences in the commons. Metroeconomica 68, 947–965.
Mármol AM, Monroy L, Caraballo MA, Zapata A (2017) Equilibria with vector-valued utilities and preference information. The analysis of a mixed duopoly. Theory and Decision 83, 365–383.

Acknowledgement: Grant PID 2021-124981NB-I00 funded by MICIU/AEI/ 10.13039/501100011033 and by "ERDF/EU", and Grant RED2022-134540-T funded by MICIU/AEI/10.13039/501100011033.