Using Composite Quality Indicators to Assess Population-Based Algorithms

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Motivation

- Multiobjective optimization problems (MOPs) are formulated to optimize simultaneously several conflicting objectives arising in many real-world situations.
- Many metaheuristic multiobjective optimization algorithms have emerged for solving MOPs, such as evolutionary algorithms [1], [2], particle swarm optimization [3], etc.
- These algorithms uses population-based techniques designed to approximate the Pareto front (PF), which is formed by all the Pareto optimal solutions, with a set of non-dominated solutions.
- A key concern in the field is how to evaluate the performance of these population-based algorithms.
  1. Convergence (closeness to the Pareto front).
  2. Spread (coverage over the Pareto front)
  3. Uniformity (distribution in the Pareto front
- Not all QIs can capture the three performance criteria (convergence, spread and uniformity).
- Population-based algorithms are ranked differently depending on the QIs used.

Objective

Building three composite QIs to globally assess the quality of the approximation sets generated by population-based multiobjective optimization algorithms.

Conceps

The composite QIs (CQIs) are build based on the double reference point (DRP) preferential scheme proposed in [4]. To normalize the value of the individual indicators :

- A value that is regarded as desirable (aspiration level).
- A value that is regarded as the limit of the acceptable (reservation level).

Different compensation degrees among the single QIs considered can be applied, which determines how bad single QI values are compensated by good ones:

- Weak DRP-based CQI (W-CQI) $\rightarrow$ Fully compensatory index
- Strong DRP-based CQI (S-CQI) $\rightarrow$ Non-compensatory index
- Mixed DRP-based CQI (M-CQI) $\rightarrow$ Partially compensatory index

The original single indicator I_{ij} is normalized as follows:

$$\text{If } I_j^{min} \leq I_{ij} \leq I_j^r, \text{ then } \hat{I}_{ij} = \frac{I_{ij} - I_j^{min}}{I_j^r - I_j^{min}}.$$
$$\text{If } I_j^r \leq I_{ij} \leq I_j^a, \text{ then } \hat{I}_{ij} = 1 + \frac{I_{ij} - I_j^r}{I_j^a - I_j^r}.$$
$$\text{If } I_j^a \leq I_{ij} \leq I_j^{max}, \text{ then } \hat{I}_{ij} = 2 + \frac{I_{ij} - I_j^a}{I_j^{max} - I_j^a}.$$

- $\hat{I}_{ij}$ is in [0,1). Original QI value is worse than the reservation level.
- $\hat{I}_{ij}$is in [1,2]. Original QI value is better than the reservation level but worse than the aspiration level.
- $\hat{I}_{ij}$is in (2,3]. Original QI value is better than the aspiration level.

Notation

Symbol	Meaning
QI	Quality indicator
CQI	Composite quality indicator
N_I	Number of quality indicators
I_j^{max}	Maximum value of quality indicator j
I_j^{min}	Minimum value of quality indicator j
I_j^a	Aspiration level of quality indicator j
I_j^r	Reservation level of quality indicator j
I_{ij}	Value of the quality indicator j for algorithm i
$\hat{I}_{ij}$	Normalized value of I_{ij} in [0, 3]
μ_j	Level of importance of quality indicator j

Proposal

- *Weak DRP-based CQI*:
  $$\text{W-CQI}_i = \sum_{j=1}^{N_I} \mu_j^w \hat{I}_{ij},$$
  The contribution of each individual indicator is relative to the total weight assigned across all indicators.
- *Strong DRP-based CQI*:
  $$\text{S-CQI}_i = \min_{j=1,\ldots,N_I} \left\{ \mu_j^s \hat{I}_{ij} \right\},$$
  Value of the worst normalized quality indicator
- *Mixed DRP-based CQI*:
  $$\text{M-CQI}_i = \lambda \cdot \text{W-CQI}_i + (1-\lambda) \cdot \text{S-CQI}_i,$$
  Partially compensatory composite quality indicator.
- Note: The DRP-based CQIs take values in the same scale as the normalized QIs, i.e. in the 0-3 range.

Experiments

- Algorithms: 18 population-based optimization algorithms: AGEMOEAII, CLIA, DGEA, EFRRR, EMyOC, GrEA, GWASFGA, HEA, LERD, LMEA, LMOCSO, MaOEADDFC, MOEADDE, NSGA-III, PPS, RVEA, SSCEA, and VaEA.
- Problems: Three-, five-, and eight-objective functions problems: DTLZ1, IDTLZ2, and WFG1 benchmark problems.
- Indicators: ER, PD, GD, Spacing
- Reference values:
  - I_j^r is set to percentile 25, for $j = 1, â, 10$.
  - I_j^a is set to percentile 75, for $j = 1, â, 10$.

  All solutions are used to compute I_j^r, I_j^a, I_j^{min}, and I_j^{max}
- 20 independent runs for each algorithm.

Results

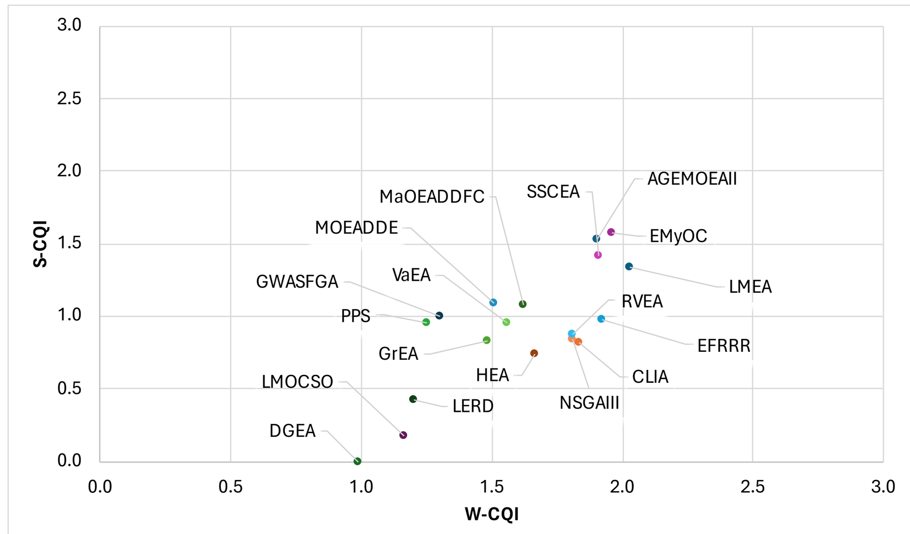

Figura: Three-objective DTLZ1 problem

Position	DTLZ1		IDTLZ2		WFG1	
	M-CQI	HV	M-CQI	HV	M-CQI	HV
1	EMyOC	EFRRR	AGEMOEAII	AGEMOEAII	AGEMOEAII	AGEMOEAII
2	AGEMOEAII	CLIA	CLIA	SSCEA	EFRRR	EFRRR
3	LMEA	RVEA	SSCEA	LMEA	NSGAIII	MaOEADDFC
4	SSCEA	NSGAIII	LMEA	EMyOC	MaOEADDFC	NSGAIII
5	EFRRR	HEA	EMyOC	CLIA	CLIA	HEA
6	MaOEADDFC	AGEMOEAII	VaEA	MaOEADDFC	HEA	CLIA
7	RVEA	SSCEA	LMOCSO	VaEA	VaEA	GWASFGA
8	CLIA	LMEA	RVEA	HEA	RVEA	VaEA
9	NSGAIII	EMyOC	MaOEADDFC	GWASFGA	SSCEA	SSCEA
10	MOEADDE	LMOCSO	GWASFGA	EFRRR	GrEA	RVEA
11	VaEA	LERD	NSGAIII	NSGAIII	GWASFGA	GrEA
12	HEA	MaOEADDFC	EFRRR	MOEADDE	LMEA	LERD
13	GrEA	VaEA	GrEA	LMOCSO	LERD	LMOCSO
14	GWASFGA	PPS	HEA	RVEA	EMyOC	LMEA
15	PPS	MOEADDE	PPS	PPS	PPS	EMyOC
16	LERD	GWASFGA	MOEADDE	GrEA	MOEADDE	PPS
17	LMOCSO	GrEA	DGEA	DGEA	LMOCSO	MOEADDE
18	DGEA	DGEA	LERD	LERD	DGEA	DGEA

Cuadro: Algorithm ranking according to the M-CQI and to the HV for three-objective problems

Theoretical results

Definition: A QI is Pareto-compliant if it satisfies that, when a non-dominated solution set strictly dominates another one, the QI value of the one dominating is strictly better than that of the dominated one.
Definition: A QI is weakly Pareto-compliant if a non-dominated solution set strictly dominates another one, the QI value of the one dominating is better or equal than that of the dominated one.
Theorem: If, at least, one Pareto-compliant single QI is combined with other weakly Pareto-compliant single QIs, the Pareto-compliance of the W-CQI and M-CQI is assured, while the S-CQI is weakly Pareto-compliant.

Conclusions

Advantages

- It can be tuned to aggregate the desired single Qis.
- It can combine information about convergence, spread and/or uniformity as wished.
- It are obtained using reservation and aspiration levels to be attained by the single QIs.
- Any population-based algorithm can be assessed with the DRP-based CQIs, an even new indicator-based algorithms can be defined according to them.

Disadvantages

- They can be time-consuming to obtain due to the computational cost of calculating the single QI values, which may increase as the number of objectives increases.

Referencias

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