A semivectorial bilevel optimization model to optimize energy prices at charging stations for electric vehicles

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Statement of the problem

- The increasing adoption of electric vehicles calls for efficient management and planning of charging infrastructure.
- Charging station operators need to set attractive but profitable prices.
- Once the prices have been set, users react to those prices by deciding where to charge.
- Users have a price threshold and may reject using the charging system if prices are too high.

Statement of the problem

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- Charging station operators need to set attractive but profitable prices.
- Once the prices have been set, users react to those prices by deciding where to charge.
- Users have a price threshold and may reject using the charging system if prices are too high.
- Bilevel optimization is a suitable framework to model this problem:
  - At the upper level, the charging stations operator fixes prices.
  - At the lower level, users decide where to charge.
- Users aim to minimize both the charging cost and the total time spent, so we have a multiobjective optimization problem at the lower level.

Some notes on bilevel optimization

$$\min_{x} \quad F(x, y)$$

subject to:

$$G_j(x, y) \leqslant 0 \quad j = 1, \ldots, q$$

where, for every fixed x, y solves:

$$\min_{y} \quad f(x, y)$$

subject to:

$$g_h(x, y) \leqslant 0 \quad h = 1, \ldots, p$$

– For every fixed x, y is an optimal solution of the lower level problem.

Some notes on bilevel optimization and multiple objectives

$$\min_{x} \quad F(x, y)$$

subject to:

$$G_j(x, y) \leqslant 0 \quad j = 1, \ldots, q$$

where, for every fixed x, y solves:

$$\min_{y} \quad \left[f_1(x, y), \ldots, f_m(x, y)\right]$$

subject to:

$$g_h(x, y) \leqslant 0 \quad h = 1, \ldots, p$$

– For every fixed x, y is an optimal solution of the lower level problem.

– For every fixed x, y is an efficient solution of the lower level problem.

– Different possibilities for solving the multiobjective problem: weighted sum method, lexicographic approach, ε-constraints method or goal programming.

Literature review

- Momber, I., Wogrin, S., & Gómez San Román, T. (2016).Retail pricing: A bilevel program for PEV aggregator secisions using indirect load control. *IEEE Transactions on Power Systems*, (1), 464–473. https://doi.org/10.1109/TPWRS.2014.2379637
  - The model is not multiobjective but the structure of the bilevel model is the same we are going to propose: the system operator at the upper level and the users at the lower level.
- González, S., Feijoo, F., Basso, F., Subramanian, V., Sankaranarayanan, S., & Das, T. K. (2022).Routing and charging facility location for EVs under nodal pricing of electricity: A bilevel model solved using special ordered set. *IEEE Transactions on Smart Grid*, (4), 3059–3068. https://doi.org/10.1109/TSG.2022.3159603
  - Multiobjective at the upper level: minimize the charging cost and minimize the travel time of electric vehicles.
- Zhang, B., Zhao, M., & Hu, X. (2023).Location planning of electric vehicle charging station with users' preferences and waiting time: Multiobjective bilevel programming model and HNSGA-II algorithm. *International Journal of Production Research*, (5), 1394–1423. https://doi.org/10.1080/00207543.2021.2023832
  - Multiobjective at the upper level: minimize the total cost of locating an sizing the charging station and minimize the service tardiness.

Sets and parameters for the mathematical formulation

Sets

- $I = \{1, \ldots, n\}$ is the set of potential users.
- $J = \{1, \ldots, m\}$ is the set of available charging stations.

Parameters

- $d_i \geqslant 0$ is the energy required by user $i \in I$.
- $b_i \geqslant 0$ is the maximum price user $i \in I$ is willing to pay per unit of energy.
- $c_{ij} \geqslant 0$ is the travel time spent by user $i \in I$ to reach charging station $j \in J$.
- $P_j \geqslant 0$ is the power installed in charging station $j \in J$.
- $C_j \geqslant 0$ is the capacity of charging station $j \in J$.

Decision variables

Decision variables

- $\pi_j \geqslant 0$ is the price per unit of energy set in charging station $j \in J$.
- $y_i \in \{0, 1\}$ takes value 1 if user $i \in I$ uses the system.
- $x_{ij} \in \{0, 1\}$ takes value 1 if user $i \in I$ goes to charging station $j \in J$.

It is enough to consider as possible prices the set of budgets $\{b_i\}_{i \in I}$.

- We define the set of indices of different budgets, $L = \{1, \ldots, |L|\}$.
- We define the ordered set different budgets, $\left\{b^1, \ldots, b^{|L|}\right\}$
- $b^{l_1} < b^{l_2}$ if $l_1 < l_2$.
- For each charging station $j \in J$ and each index $l \in L$, we define the variables:

$$v_j^l = \begin{cases} 1 & \text{if charging station } j \text{ is priced at } b^l \\ 0 & \text{otherwise} \end{cases}$$

- We define a map $\sigma \colon I \longrightarrow L$: $\sigma(i) = l$ if user i has a budget equal to b^l.

Mathematical formulation of the model

The total amount of money paid by the users is:

$$\sum_{i \in I} \sum_{j \in J} \pi_j d_i x_{ij}$$

By the definition of v-variables it is true that, for every $j \in J$:

$$\pi_j = \left(\sum_{l=1}^{|L|} b^l v_j^l \right)$$

Thus,

$$\sum_{i \in I} \sum_{j \in J} \pi_j d_i x_{ij} = \sum_{i \in I} d_i \sum_{j \in J} \left(\sum_{l=1}^{|L|} b^l v_j^l \right) x_{ij}$$

Mathematical formulation of the model

The upper level problem is:

$$\max_{v,y} \quad \sum_{i \in I} d_i \sum_{j \in J} \left(\sum_{l=1}^{|L|} b^l v_j^l \right) x_{ij}$$

s.t.:

$$\sum_{l=1}^{|L|} v_j^l = 1 \qquad j \in J$$

$$y_i \geqslant \sum_{l=1}^{\sigma(i)} v_j^l \qquad i \in I \quad j \in J$$

$$y_i \leqslant \sum_{j \in J} \sum_{l=1}^{\sigma(i)} v_j^l \qquad i \in I$$

$$v_j^l \in \{0,1\} \qquad j \in J \quad l \in L$$

$$y_i \in \{0,1\} \qquad i \in I$$

Mathematical formulation of the model

The lower level problem is:

$$\min_{x} \quad \left\{ \begin{array}{l} \displaystyle\sum_{i\in I} d_i \sum_{j\in J} \left(\sum_{l=1}^{|L|} b^l v_j^l \right) x_{ij} \\ \\ \text{“Total time spent for charging”} \end{array} \right\}$$

s.t.:

$$\sum_{j\in J} x_{ij} = y_i \qquad i \in I$$

$$x_{ij} \leqslant \sum_{l=1}^{\sigma(i)} v_j^l \qquad i \in I \quad j \in J$$

$$\sum_{i\in I} d_i x_{ij} \leqslant C_j \quad j \in J$$

$$x_{ij} \in \{0, 1\} \qquad i \in I \quad j \in J$$

Wardrop's equilibrium

- Wardrop's equilibrium is a concept used to describe the behaviour of multiple users that share limited resources in a system.

- It is typically used in problems related to road traffic on transport networks.

- Wardrop's equilibrium is attained when any user can reduce their travel time by unilaterally changing their route.

  1. First Wardrop's principle (user equilibrium): The travel times of the routes used by each user are equal or less equal to the travel times of the available alternatives routes.

  2. Second Wardrop's principle (system equilibrium): The traffic organization is made in such a way that the total cost of the system is minimized.

- Some properties:
  - Selfish behaviour.
  - In the equilibrium, users has not the incentive to change their decision.

How do we measure the time spent in the system?

- The fixed time spent by a user i when choosing a charging station j is:

$$\underbrace{c_{ij}}_{\text{travel time}} + \underbrace{\frac{d_i}{P_j}}_{\text{charging time}}$$

- The amount of users going to a charging station j increases the time spent by the users: the more users, the longer the time due to congestion and waiting time.

- Being f_j the "flow" of users in charging station $j \in J$:

$$\underbrace{\beta_j f_j^2}_{\text{waiting time}} = \beta_j \left(\sum_{i \in I} x_{ij} \right)^2$$

- The total time spent in the system is:

$$\sum_{j \in J} \left[\sum_{i \in I} c_{ij} x_{ij} + \sum_{i \in I} \frac{d_i}{P_j} x_{ij} + \beta_j \left(\sum_{i \in I} x_{ij} \right)^2 \right]$$

How do we measure the time spent in the system?

- $\sum_{j \in J} \left[\sum_{i \in I} c_{ij} x_{ij} + \sum_{i \in I} \frac{d_i}{P_j} x_{ij} + \beta_j \left(\sum_{i \in I} x_{ij} \right)^2 \right]$ can be linearized.
  - $\left(\sum_{i \in I} x_{ij} \right)^2 = \ldots = \sum_{i=1}^{n} x_{ij}^2 + 2 \sum_{1 \leqslant i < k \leqslant m} x_{ij} x_{kj}$
  - $x_{ij}^2 = x_{ij}$ due to the binary character of x_{ij}.
  - Products $x_{ij} x_{kj}$ can be linearized by introducing variables $\gamma_{(ik)j} = x_{ij} x_{kj} \in \{0, 1\}$ and the set of constraints:

$$
\begin{array}{lll}
\gamma_{(ik)j} \leqslant x_{ij} & i, k \in I \text{ with } i < k & j \in J \\
\gamma_{(ik)j} \leqslant x_{kj} & i, k \in I \text{ with } i < k & j \in J \\
\gamma_{(ik)j} \geqslant x_{ij} + x_{kj} - 1 & i, k \in I \text{ with } i < k & j \in J
\end{array}
$$

  - $\left(\sum_{i \in I} x_{ij} \right)^2 = \sum_{i \in I} x_{ij} + 2 \sum_{i \in I} \sum_{\substack{k \in I \\ i < k}} \gamma_{(ik)j}$

A matheuristic algorithm to solve the problem

The algorithm combines:

- An evolutionary algorithm is used to explore the feasible set of the upper level decision variables.
- The chromosomes are vectors of length $|J|$ and encode vectors of prices:

$$\underbrace{\boxed{1}\,\boxed{7}\,\boxed{3}\,\boxed{|L|}\;\cdots\;\boxed{2}\,\boxed{6}\,\boxed{1}\,\boxed{1}}_{|J| \text{ components}}$$

- The lower level problem is solved exactly using the weighted sum method.
- Two fitness values are computed per chromosome:
  - $w_1 = 0.1$ and $w_2 = 0.9$: prioritizes minimizing time.
  - $w_1 = 0.9$ and $w_2 = 0.1$: prioritizes minimizing cost.
- Is it better to solve the non-linear lower level problem or the linearized one?

A matheuristic algorithm to solve the problem

- The generation of the initial population is totally random.
- The uniform crossover operator is used to generate children from the set of parents.
- The mutation operator alters each gen independently with a probability of mutation $p_m = \frac{1}{|J|}$.
- The population is subdivides in two groups:
  - Subgroup 1: guided by the "optimistic" fitness.
  - Subgroup 2: guided by the "pessimistic" fitness.
- The selection of survivors is made according to the two subgroups of population.

Conclusions and further research

- Conclusions:
  - We have exploited the hierarchical nature of the problem to formulate a bilevel model that is conceptually correct and truly reflects the interaction between the charging station operator and the users.
  - Users are modelled as multiobjective decision makers, considering both the price paid and the time spent.
  - We are testing the proposed matheuristic algorithm to solve the model.
- Additional considerations for further research:
  - Introduce periods of time.
  - Incorporate a realistic price structure: in a real situation it does not make sense for a charging station with more power to be cheaper than one with less power.
  - Explore other methods, such as ε-constraints or goal programming, to solve the multiobjective optimization problem at the lower level.

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