Ordinal Rank-based Preference Elicitation Model For Assigning Weights to Decision Criteria

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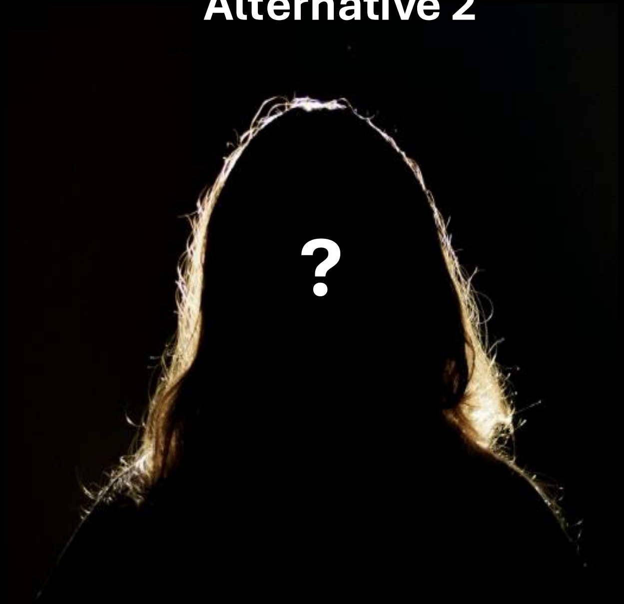
Preference among criteria: A dating example

Alternative 1

Alternative 2



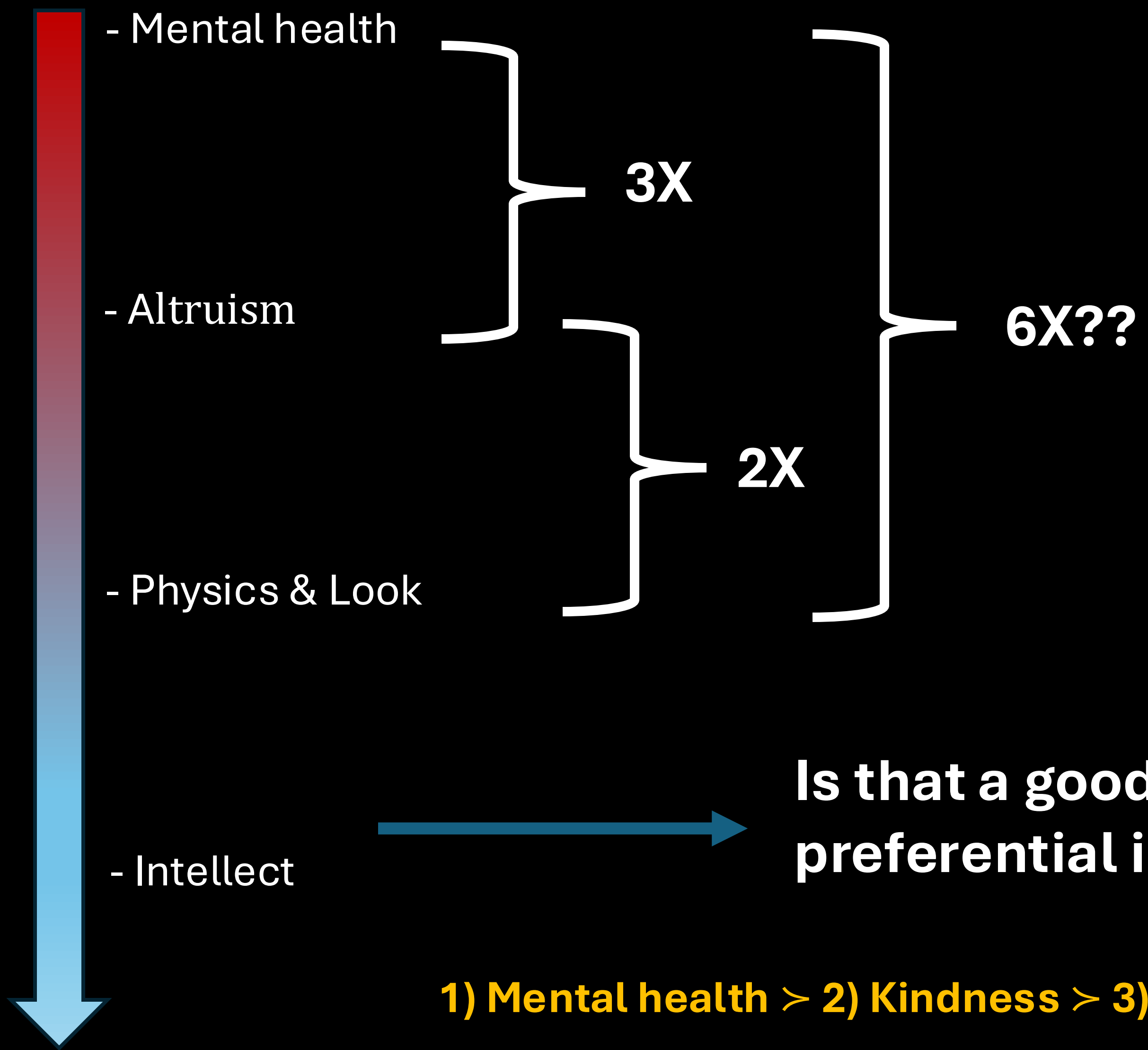
Objective:

Maximizing the chance of a healthy partnership that exceeds 5 years

Preference on Selection Criteria:

1) Mental health $\succ$ 2) Altruism $\succ$ 3) Physics & Look $\succ$ 4) Intellect

Ordinal Preference Elicitation

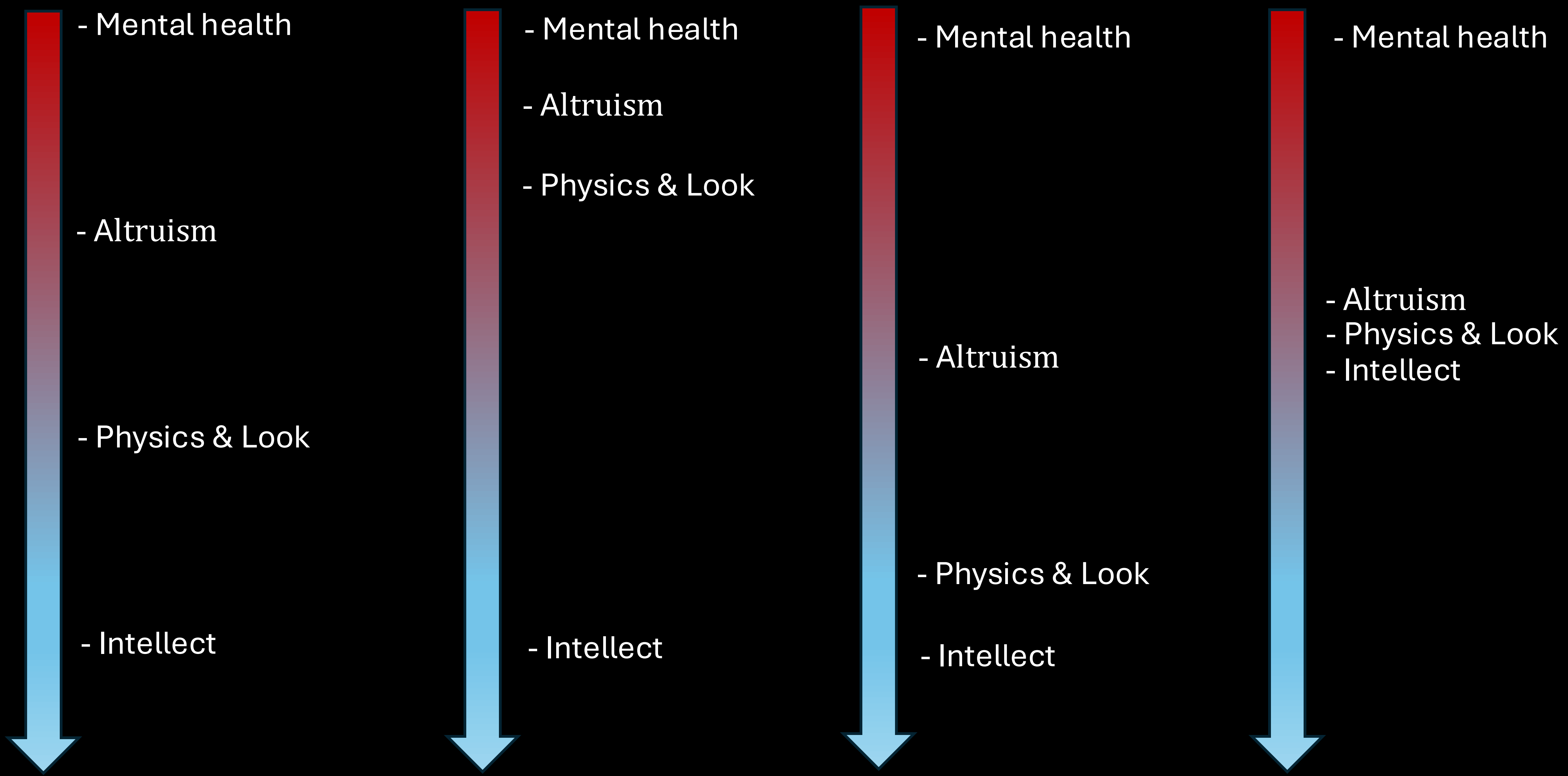


- Intransitivity
- Non-linearity
- Cardinality

Is that a good representation of my preferential information

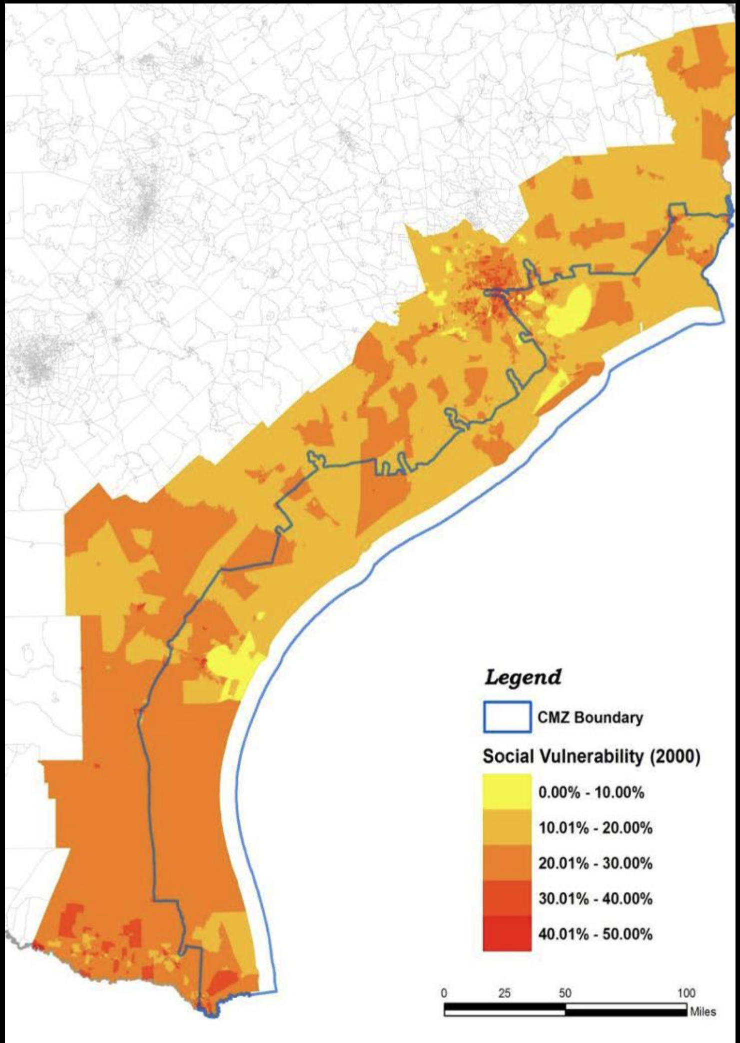
1) Mental health ≻ 2) Kindness ≻ 3) Physics & Look ≻ 4) Intellect

1) Mental health ≻ 2) Kindness ≻ 3) Physics & Look ≻ 4) Intellect



Zonning urban areas for disaster Social Vulnerability

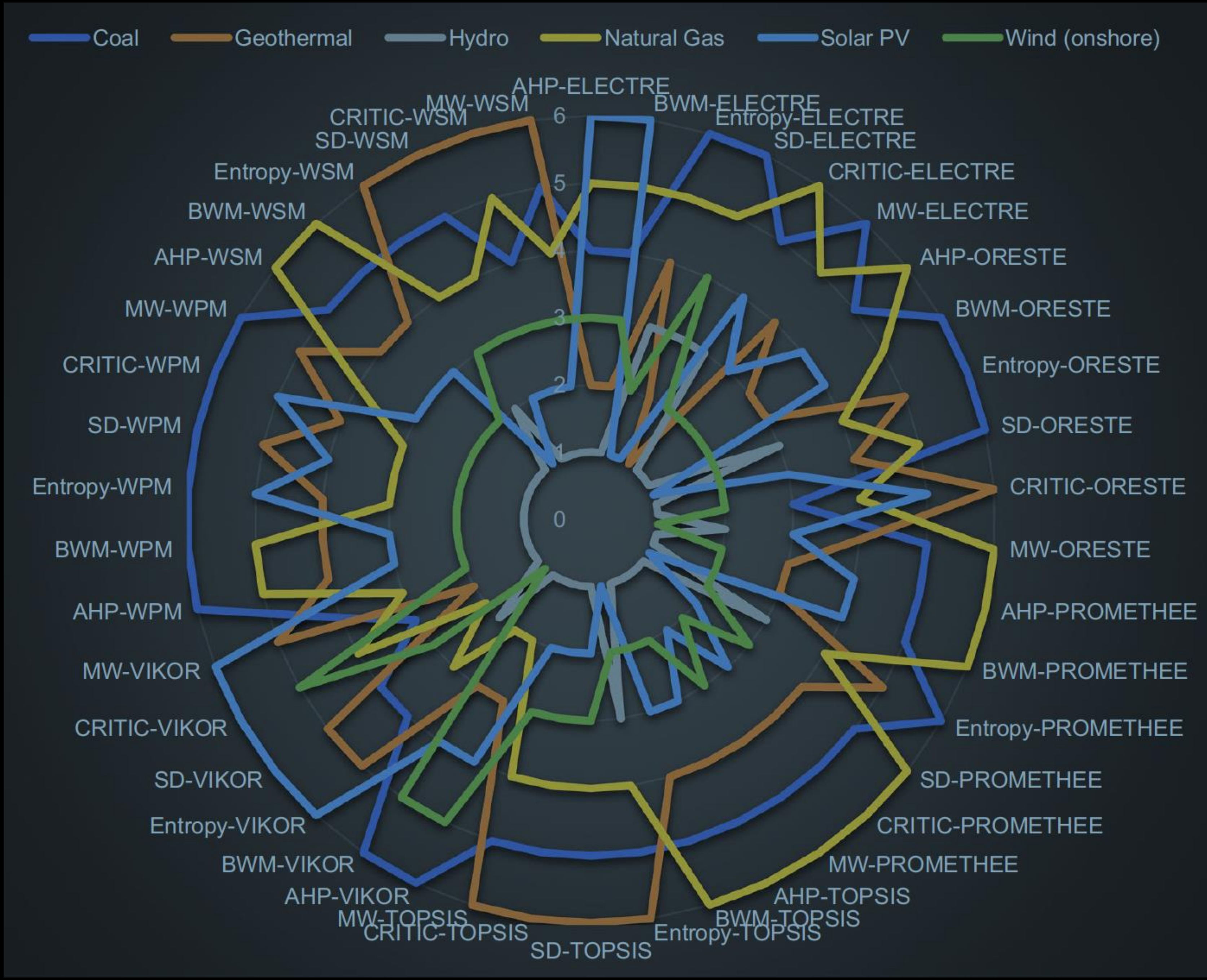
Base Social Vulnerability Indicators (percentages)	2nd Order	3rd Order
1. Single parent households with children/Total Households	Potential Child care Needs	Socially Vulnerable Hotspot
2. Population 5 or below/Total Population		
3. Population 65 or above/Total Population	Potential Elder Care Needs	
4. Population 65 or above & below poverty/Pop. 65 or above		
5. Workers using public transportation/Civilian pop. 16+ and employed	Potential Trans. needs	
6. Occupied housing units without a vehicle/Occupied housing units (HUs)		
7. Occupied Housing units/Total housing units	Potential Housing Needs (Temporary Shelter and housing recovery)	
8. Persons in renter occupied housing units/Total occupied housing units		
9. Non-white population/Total population		
10. Population in group quarters/Total population		
11. Housing units built 20 years ago/Total housing Units		
12. Mobile Homes/Total housing units		
13. Persons in poverty/Total population		
14. Occupied housing units without a telephone/Total occupied HUs	Potential Civic Capacity needs	
15. Population above 25 with less than high school/Total pop above 25		
16. Population 16+ in labor force and unemployed/Pop in Labor force 16+		
17. Population above 5 that speak English not well or not at all/Pop > 5		



A Comparative Analysis of Precise Methods

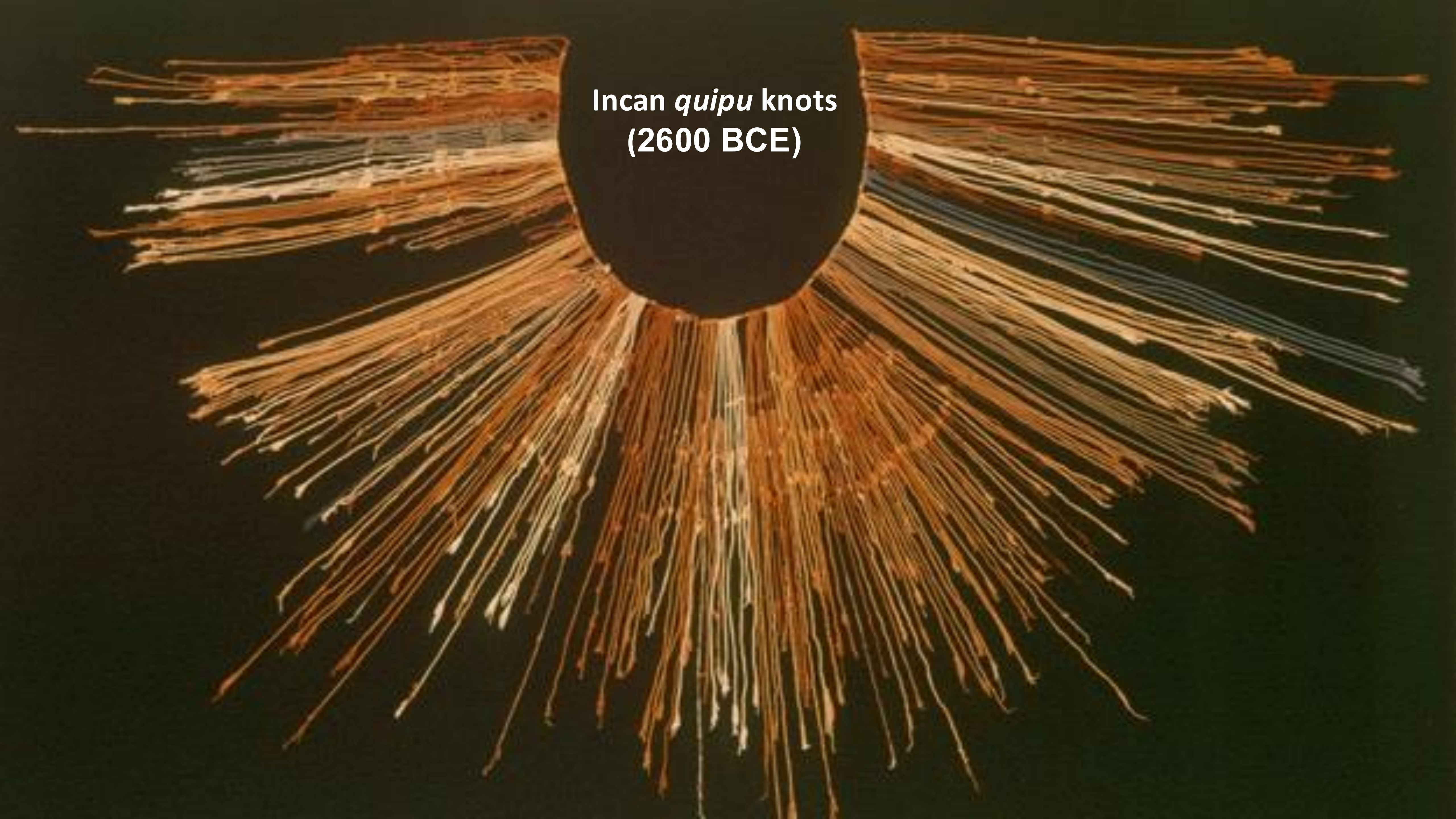
42 different models

Şahin 2020



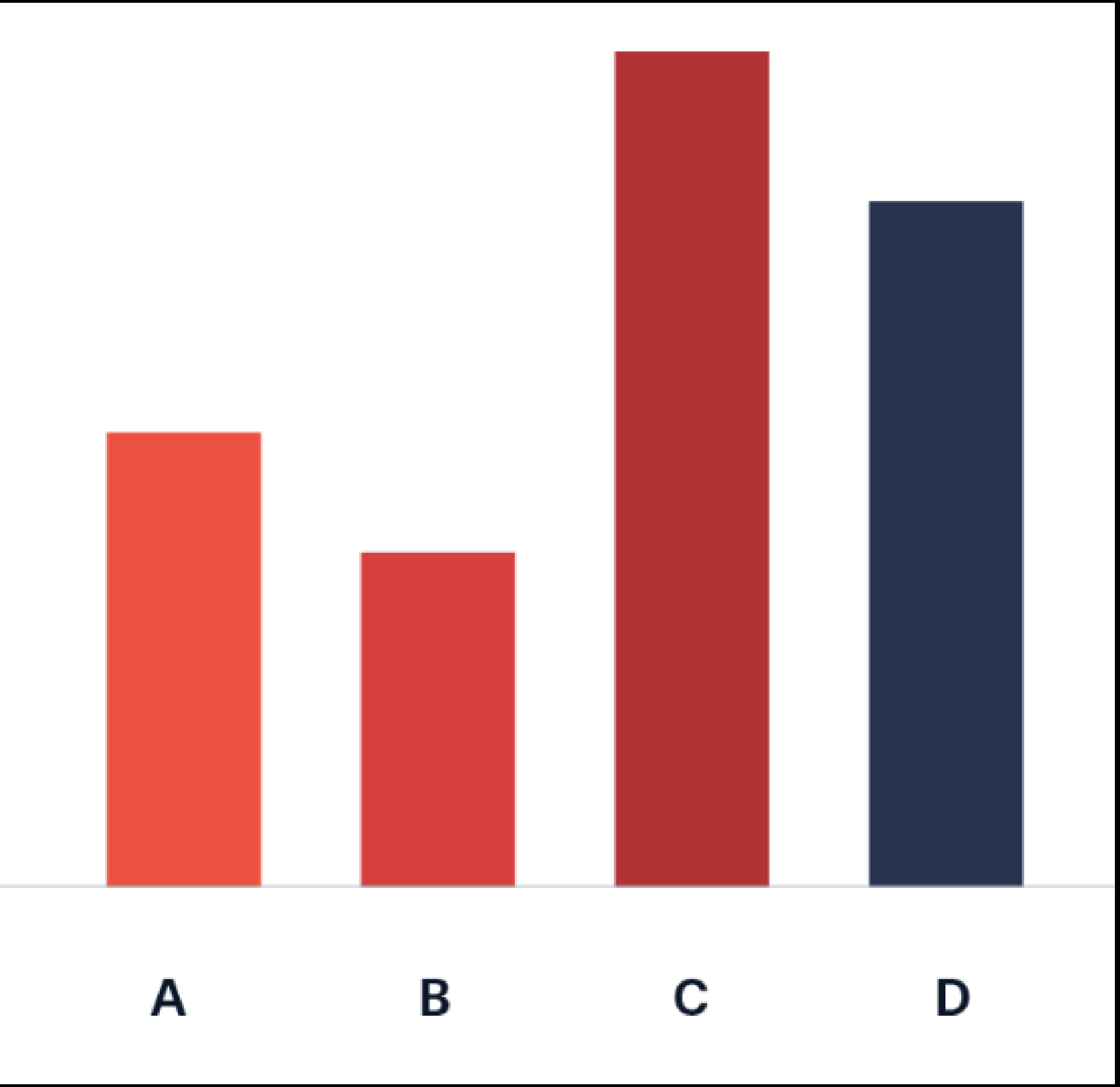
Which one is more?

Can you understand the quantity?

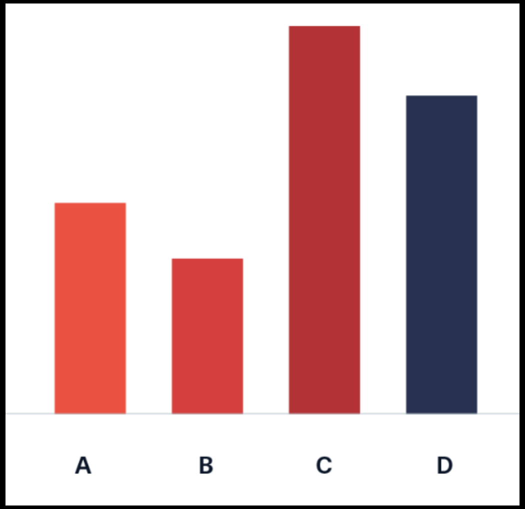


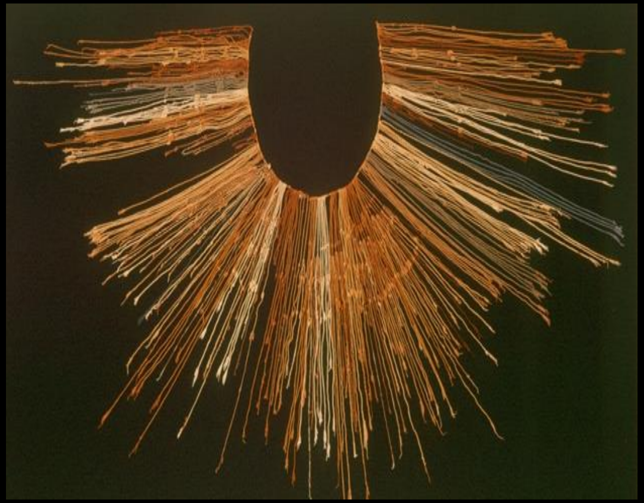
Incan *quipu* knots
(2600 BCE)

Which variable has the higher share?



What do they all have in common?

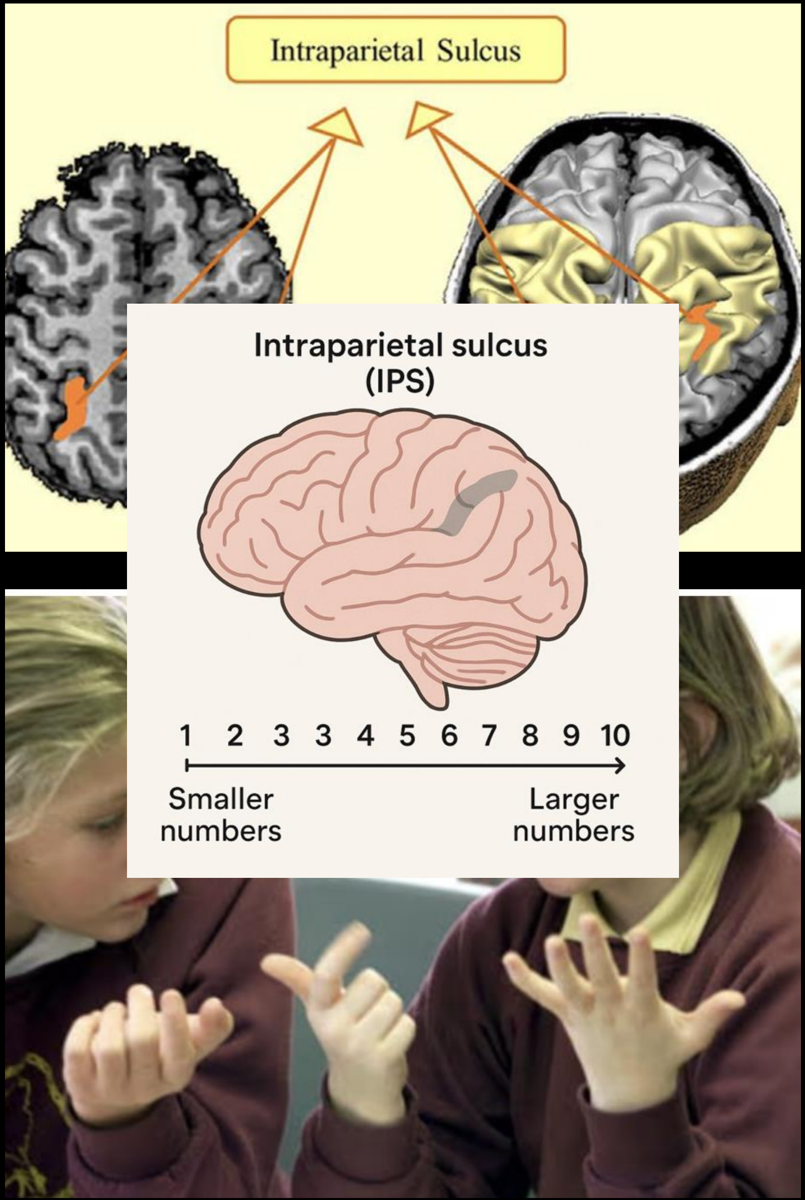




Did you need numbers to understand the relative magnitude?

Spatial-numerical association & Intraparietal Sulcus (IPS)

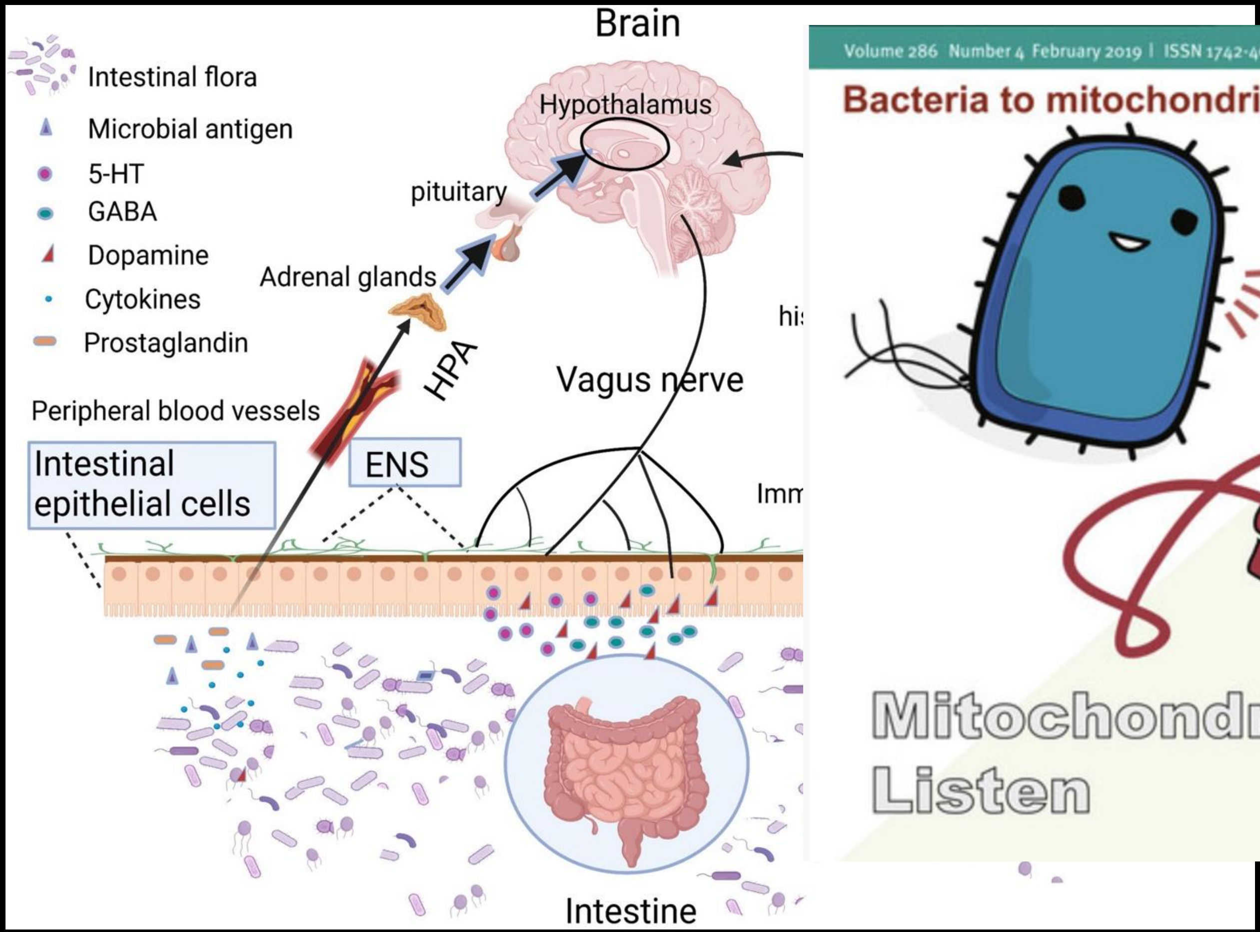
- Human cognition does not naturally express preferences with numbers.
- **IPS:** Responsible for **visuo-spatial function**: Activated to understand numbers!
- Also to count: The exact part of the brain responsible for counting on fingers
- Humans tend to mentally represent numbers along a horizontal line
- **We have a mental image of numbers in the sense of distance: We don't count, WE SEE quantities!**

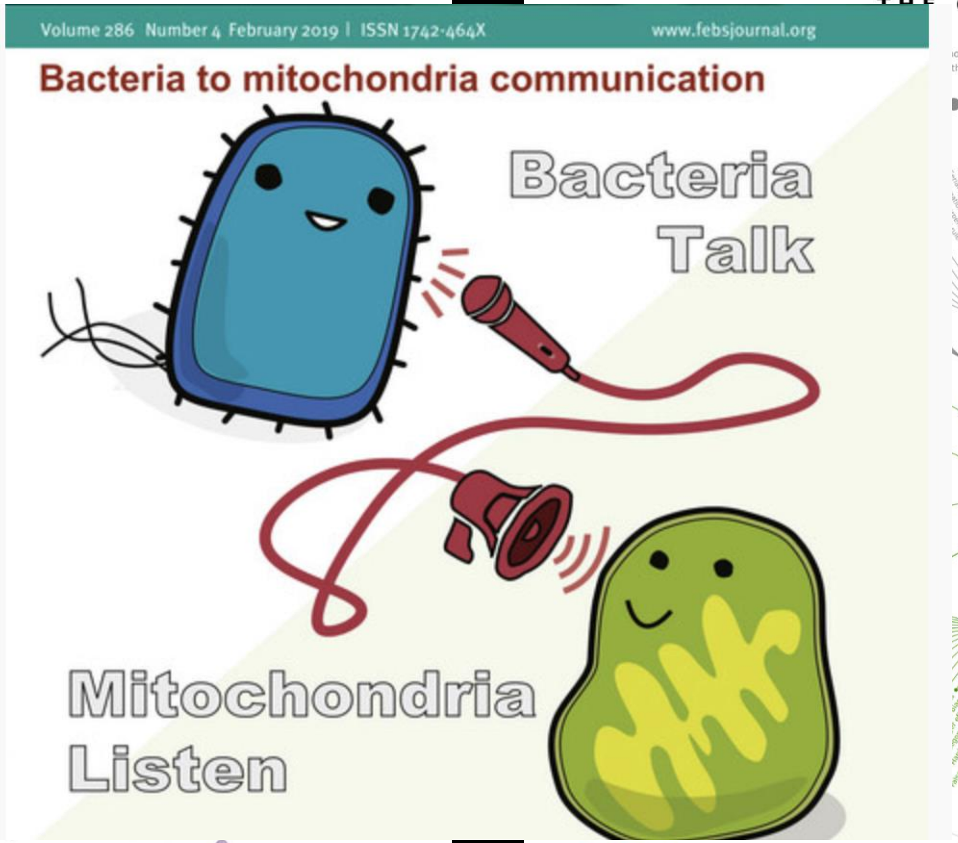


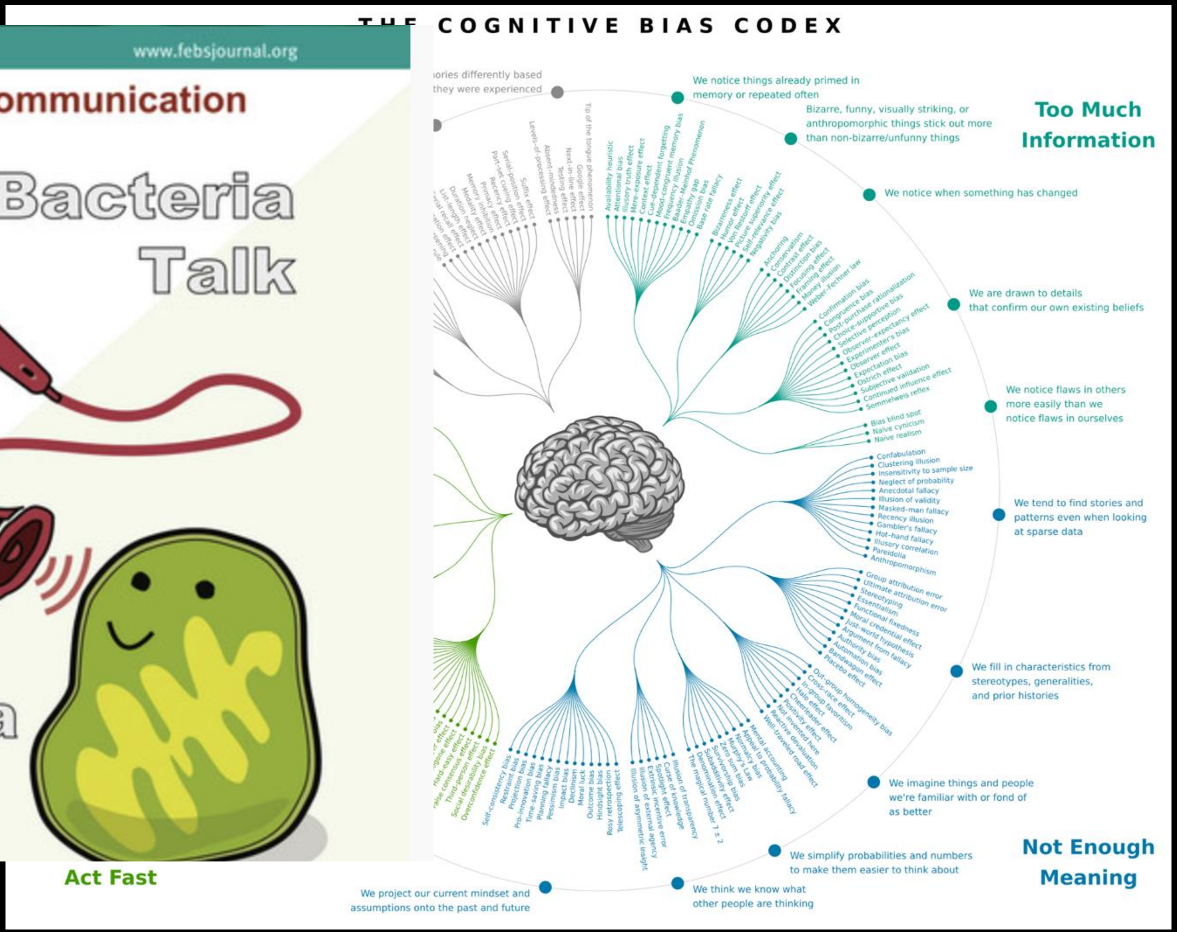
1- Is our preference a rational choice?
2- Does expressing it in numbers make it less subjective?

Gut-Brain Axis

Cognitive biases







Zhu et al., 2022

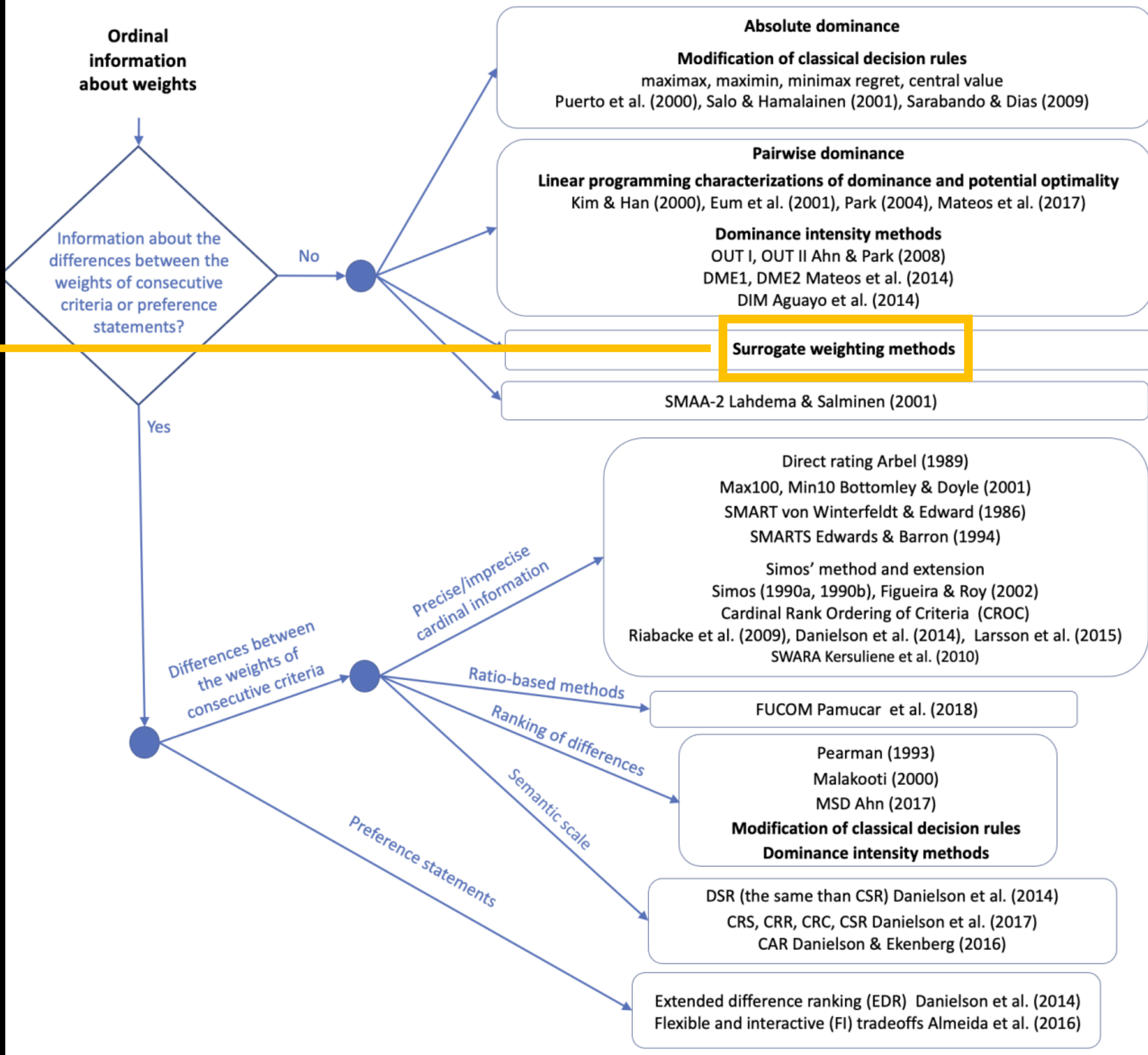
Han et al., 2019

Imprecise-surrogate weight allocation approach

- Doesn't force DMs to express the exact numerical value or verbal representation of preference beyond their cognitive capacity
- DMs are both more confident and comfortable with ranking criteria than assigning numerical values or verbal scale *(Barfod and Leleur (2014); Danielson and Ekenberg, 2016))*
- Operates based on minimal data

1. They **reduce cognitive burden** (more natural)
2. They are **more robust under limited/inconsistent data**.
3. They avoid false precision.

Equal weights (EW)	$w_i = \frac{1}{n}, i = 1, ..., n$
Rank sum (RS)	$w_i = \frac{n+1-i}{\sum_{j=1}^{n}(n+1-j)} = \frac{2(n+1-i)}{n(n+1)}, i = 1, ..., n$
Rank exponent (RE)	$w_i = \frac{(n+1-i)^p}{\sum_{j=1}^{n}(n+1-j)^p}, i = 1, ..., n$
Rank reciprocal (RR)	$w_i = \frac{1/i}{\sum_{j=1}^{n} 1/j}, i = 1, ..., n$
Rank-order centroid (ROC)	$w_i = \frac{\sum_{j=i}^{n} 1/j}{n}, i = 1, ..., n$
Equal ratio fixed (ERF)	$w_i(r) = \frac{r^i}{\sum_{j=1}^{n} r^j}, i = 1, ..., n$
Geometric weights (GW)	$w_i = \frac{100}{(\sqrt{2})^{i-1}}, i = 1, ..., n$
Variable-slope linear (VSL)	$w_i = \frac{1-(3.19514+\frac{37.75756}{n})(\frac{i-1}{100})}{\sum_{j=1}^{n}(1-(3.19514+\frac{37.75756}{n})(\frac{j-1}{100}))}, i = 1, ...$
Maximum entropy ordered (MEO)	$w_i = \sqrt[n-1]{w_1^{n-i} * w_n^{i-1}}, i = 2, ..., n-1$
Weighted averaging (MEOWA)	w_1, w_n predetermined
Least-squared ordered weighted Averaging (LSOWA)	$w_i(\theta) = \frac{1}{n} + \frac{6(2i-n-1)}{n(n+1)} + (\frac{1}{2} - \theta)\ i = 1,$
Sum reciprocal (SR)	$w_i = \frac{1/i+\frac{n+1-i}{n}}{\sum_{j=1}^{n}(1/j+\frac{n+1-j}{n})}, i = 1, ..., n$
Rank order total (ROT)	$w_i = \frac{3(n+1-i)(n+2-i)}{n(n+1)(n+2)}, i = 1, ..., n$
Geometric sum (GS)	$w_i = \frac{s^{i-1}}{\sum_{j=1}^{n} s^{j-1}}, i = 1, ..., n$
Generalized rank sum (GRS)	$w_i(\delta) = \frac{(\delta-1)\delta^{n-i}}{\delta^n-1}, i = 1, ..., n$
Improved ROC (IROC)	$w_i(\varphi_{in}, ..., \varphi_{nn}) = \sum_{j=i}^{n}(\varphi_{jn}/j), i = 1, ..., n$
Generalized ROC (GROC)	$w_i(\varphi) = \sum_{j=i}^{n} \frac{\varphi^{n-j}}{n \sum_{k=1}^{n-j-i} \varphi^{k-1}}, i = 1, ..., n$
Rank order logarithm (ROL)	$w_i = \frac{\log(\frac{i}{n+1})}{\sum_{j=i}^{n} \log(\frac{j}{n+1})}, i = 1, ..., n$



Chergui, 2024

Gaps of existing methods

1- Nonlinearity and cardinality

2- Equally important criteria

3- Marginalized lower criteria problem

4- Equalized weights problem

5- Flexible for DM with uncertainty of decision environment

Roszkowska,, 2013

Approximations for criteria weights given by used different formulas, in ca
of n=2,…, 7 criteria

Number of criteria	Rank ordering methods			
	Centroid weight (*ROC*)	Reciprocal weight (*RR*)	Rank Sum weight (*RS*)	Equal weight (*EW*)
n=2	w_1=0.75 w_2=0.25	w_1=0.67 w_2=0.33	w_1=067 w_2=0.33	w_1=1/2 w_2=1/2
n=3	w_1=0.62 w_2=0.28 w_3=0.12	w_1=0.55 w_2=027 w_3=0.18	w_1=0.50 w_2=0.33 w_3=0.17	w_1=1/3 w_2=1/3 w_3=1/3
n=4	w_1=0.52 w_2=0.27 w_3=0.15 w_4=0.06	w_1=0.48 w_2=0.24 w_3=0.16 w_4=0.12	w_1=0.40 w_2=0.30 w_3=0.20 w_4=0.10	w_1=1/4 w_2=1/4 w_3=1/4 w_4=1/4
n=5	w_1=0.45 w_2=0.26 w_3=0.16 w_4=0.09 w_5=0.04	w_1=0.44 w_2=0.22 w_3=0.14 w_4=0.11 w_5=0.09	w_1=0.33 w_2=0.27 w_3=0.21 w_4=0.12 w_5=0.07	w_1=1/5 w_2=1/5 w_3=1/5 w_4=1/5 w_5=1/5
n=6	w_1=0.41 w_2=0.24 w_3=0.16 w_4=0.10 w_5=0.06 w_6=0.03	w_1=0.41 w_2=0.21 w_3=0.13 w_4=0.10 w_5=0.08 w_6=0.07	w_1=0.29 w_2=0.24 w_3=0.19 w_4=0.14 w_5=0.09 w_6=0.05	w_1=1/6 w_2=1/6 w_3=1/6 w_4=1/6 w_5=1/6 w_6=1/6
n=7	w_1=0.37 w_2=0.23 w_3=0.16 w_4=0.11 w_5=0.07 w_6=0.04 w_7=0.02	w_1=0.39 w_2=0.19 w_3=0.13 w_4=0.09 w_5=0.08 w_6=0.07 w_7=0.05	w_1=0.25 w_2=0.21 w_3=0.18 w_4=0.14 w_5=0.11 w_6=0.07 w_7=0.04	w_1=1/7 w_2=1/7 w_3=1/7 w_4=1/7 w_5=1/7 w_6=1/7 w_7=1/7

Step	Task	Mathematical representation
1	Input Data: The model begins with an ordinal ranking	for criteria i is $d_i \wedge d_1 = 0$ where it ... fore we have $d_1 \leq$ ... $d_2 \geqslant d_3 \geqslant \cdots d_n$
2	Calculate the Mean Rank ($\bar{d}$): The mean rank is computed as the average of all given ranks.	$\bar{d} = (\Sigma\, d_i) / n$
3	Compute the Central Tendency Measure (v_o): This ste... no...	$\sqrt{\frac{\sum^{n}(d_i-\bar{d})^2}{\ldots}} + d_{max}$
4	Group Tied Criteria (G_j): Criteria with identical ranks are grouped together to ensure consistent weight distribution.	Identify groups $G_1, G_2, \ldots, G_k$ such that all criteria in group G_j have the same d_i value.
5	Determine Intermediate Weights ($\hat{d}_i$): A logarithmic transformation and a **confidence score (β)** are ... maintaining stability	$\hat{d}_i = \lvert d_i - v_o \rvert \times (1 + \beta) \times \ln(\lvert d_i - v_o \rvert + c)\,,\ \forall\, i \in G_j$
6	Calculate Final Weights (w_i): Weights are normalized by summing over all group contributions and scaling to a percentage	$w_i = (\hat{d}_i / \Sigma\,(\lvert G_j \rvert \times \hat{d}_j)) \times 100, \forall\, i \in G_j$ $\sum^{n} w_i = 1$

- **Enough non-linearity:** To still capture the relative importance of higher-ranked criteria.
- **Not too extreme:** To avoid marginalizing lower-ranked criteria.

c a regularization parameter to control the behaviour of the natural logarithm function near zero

β : linear scaling factor dampen/amplify the impact of the logarithmic transformation

Uncertain decision-makers (β = -0.5)

Ordinal weights of 2 to 10 attributes										
n	w_1 (%)	w_2 (%)	w_3 (%)	w_4 (%)	w_5 (%)	w_6 (%)	w_7 (%)	w_8 (%)	w_9 (%)	w_{10} (%)
2	61.5	38.5								
3	42.1	31.6	26.3							
4	32.5	26.8	22.8	17.9						
5	26.2	23.5	20.5	17	12.8					
6	21.9	20.4	18.2	15.5	12.5	11.5				
7	18.8	18	16.5	14.6	12.5	10.8	8.8			
8	16.5	16	15	13.6	12	10.4	8.8	7.5		
9	14.8	14.5	13.8	12.7	11.5	10.1	8.8	7.5	6.3	
10	13.5	13.3	12.8	12	11	9.9	8.8	7.6	6.4	5.5

Moderately confident decision-makers (β = 0)

Ordinal weights of 2 to 10 attributes										
n	w_1 (%)	w_2 (%)	w_3 (%)	w_4 (%)	w_5 (%)	w_6 (%)	w_7 (%)	w_8 (%)	w_9 (%)	w_{10} (%)
2	70.7	29.3								
3	52.6	31.6	15.8							
4	42.9	28.6	19	9.5						
5	36.4	27.3	18.2	12.1	6.1					
6	31.6	26.3	21.1	15.8	5.3	0				
7	28	24	20	16	8	4	0			
8	25	23.4	21.9	15.6	9.4	3.1	1.6	0		
9	22.9	22.9	20	17.1	11.4	4.3	1.4	0	0	
10	21	21	20	17	11	5	3	1	0	0

Highly confident decision-makers (β = 1)

Ordinal weights of 2 to 10 attributes										
n	w_1 (%)	w_2 (%)	w_3 (%)	w_4 (%)	w_5 (%)	w_6 (%)	w_7 (%)	w_8 (%)	w_9 (%)	w_{10} (%)
2	80.4	19.6								
3	60.2	30.1	9.7							
4	48.3	32.2	16.1	3.4						
5	40.2	30.1	20.1	6.7	2.9					
6	34.4	28.7	21.5	10.7	3.6	1.1				
7	30.1	26.9	21.6	13	6.2	2	0.2			
8	26.8	25.1	21.5	14.5	8.1	3.3	0.8	0		
9	24.2	23.6	21.2	15.3	9.6	4.6	1.6	0.3	0	
10	22.1	22.3	20.7	15.6	10.7	5.8	2.5	0.6	0.1	0

Weight for the first 3 ranked criteria with different confidence factors

(β = 1.00)

n	w_1 (%)	w_2 (%)	w_3 (%)
2	80.4	19.6	
3	60.2	30.1	9.7
4	48.3	32.2	16.1
5	40.2	30.1	20.1
6	34.4	28.7	21.5
7	30.1	26.9	21.6
8	26.8	25.1	21.5
9	24.2	23.6	21.2
10	22.1	22.3	20.7

(β = 0.00)

n	w_1 (%)	w_2 (%)	w_3 (%)
2	70.7	29.3	
3	52.6	31.6	15.8
4	42.9	28.6	19
5	36.4	27.3	18.2
6	31.6	26.3	21.1
7	28	24	20
8	25	23.4	21.9
9	22.9	22.9	20
10	21	21	20

(β = -0.5)

n	w_1 (%)	w_2 (%)	w_3 (%)
2	61.5	38.5	
3	42.1	31.6	26.3
4	32.5	26.8	22.8
5	26.2	23.5	20.5
6	21.9	20.4	18.2
7	18.8	18	16.5
8	16.5	16	15
9	14.8	14.5	13.8
10	13.5	13.3	12.8

A quick comparison:

n	w_1 (%)	w_2 (%)	w_3 (%)	w_4 (%)	w_5 (%)	w_6 (%)	w_7 (%)
2	61.5	38.5					
3	42.1	31.6	26.3				
4	32.5	26.8	22.8	17.9			
5	26.2	23.5	20.5	17	12.8		
6	21.9	20.4	18.2	15.5	12.5	11.5	
7	18.8	18	16.5	14.6	12.5	10.8	8.8

Rank ordering methods		
Reciprocal weight (*RR*)	Rank Sum weight (*RS*)	Equal weight (*EW*)
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w_1=0.55 w_2=027 w_3=0.18	w_1=0.50 w_2=0.33 w_3=0.17	w_1=1/3 w_2=1/3 w_3=1/3
w_1=0.48 w_2=0.24 w_3=0.16 w_4=0.12	w_1=0.40 w_2=0.30 w_3=0.20 w_4=0.10	w_1=1/4 w_2=1/4 w_3=1/4 w_4=1/4
w_1=0.44 w_2=0.22 w_3=0.14 w_4=0.11 w_5=0.09	w_1=0.33 w_2=0.27 w_3=0.21 w_4=0.12 w_5=0.07	w_1=1/5 w_2=1/5 w_3=1/5 w_4=1/5 w_5=1/5
w_1=0.41 w_2=0.21 w_3=0.13 w_4=0.10 w_5=0.08 w_6=0.07	w_1=0.29 w_2=0.24 w_3=0.19 w_4=0.14 w_5=0.09 w_6=0.05	w_1=1/6 w_2=1/6 w_3=1/6 w_4=1/6 w_5=1/6 w_6=1/6
w_1=0.39 w_2=0.19 w_3=0.13 w_4=0.09 w_5=0.08 w_6=0.07 w_7=0.05	w_1=0.25 w_2=0.21 w_3=0.18 w_4=0.14 w_5=0.11 w_6=0.07 w_7=0.04	w_1=1/7 w_2=1/7 w_3=1/7 w_4=1/7 w_5=1/7 w_6=1/7 w_7=1/7

Short about me:

- Teaching: Sociology of Disasters, Complex Adaptive Systems
- UN advisor for the disaster resilience and climate change adaptation projects
- Research profile: Disaster Resilience and Risk Management
- PhD in Complex Civil Systems: decision analysis and post-disaster infrastructure network
- Engineer by training !!
- Informal research interest: Neuroendocrinology and evolutionary biology
- What am I doing here? Because the MCDM community is very welcoming and forgiving!!!

Thank you very much for your attention

Questions?

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